

RHODES UNIVERSITY
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Lecture Notes CCR

AM4.4 - Geometric Control

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“Where there is matter, there is geometry”.

J. KEPLER

“Imagination is more important than knowledge”.

A. EINSTEIN

“Knowledge is power”.

F. BACON

“Power is nothing without control”.

ANONYMUS

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