

RHODES UNIVERSITY
Grahamstown 6140, South Africa

Lecture Notes

CCR

M4.3 - Geometry

CLAUDIU C. REMSING

DEPT. of MATHEMATICS (Pure and Applied)

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“Where there is matter, there is geometry”.

J. KEPLER

“Imagination is more important than knowledge”.

A. EINSTEIN

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