

Appendix A

Answers and Hints to Selected Exercises

Propositions and Predicates

1. FALSE. (The negation of an implication is *not* an implication.)
2. $FTT \quad FF$.
3. TTT .
4. (a) Always FALSE.
(b) Always TRUE.
(c) Always TRUE.
(d) Always TRUE.
(e) TRUE *only when* $\tau(p) = \tau(q) = 0$ or $\tau(p) = \tau(q) = 1$.
(f) FALSE *only when* $\tau(p) = 0$ and $\tau(q) = 1$.
(g) Always TRUE.
(h) Always TRUE.
(i) FALSE *only when* $\tau(p) = \tau(r) = 1$ and $\tau(q) = 0$.
(j) FALSE *only when* $\tau(p) = \tau(r) = 1$ and $\tau(q) = 0$.
5. Observe that $RHS = \tau(\neg(p \leftrightarrow q))$.

8. When $\tau(p) = \tau(r) = 0$, then $\tau((p \rightarrow q) \rightarrow r) = 0$ and $\tau(p \rightarrow (q \rightarrow r)) = 1$.

9. (a), (c), (k).

10. (c), (d).

13. (a) $\forall x F(x, Bob)$.

(b) $\forall y F(Kate, y)$.

(c) $\forall x \exists y F(x, y)$.

(d) $\neg \exists x \forall y F(x, y)$.

(e) $\forall y \exists x F(x, y)$.

(f) $\neg \exists x (F(x, Fred) \wedge F(x, Jerry))$.

(g) $\neg \exists x F(x, x)$.

14. $FTF \quad FTT \quad TFF$.

15. (a) $\exists x (x^2 + 2x - 3 \neq 0)$: TRUE.

(b) $\forall x (x^2 - 2x + 5 > 0)$: TRUE.

(c) $\exists x \forall r (xr \neq 1)$: TRUE.

(d) $\exists x \forall m (x^2 \geq m)$: FALSE.

(e) $\forall m \exists x (x^2 \geq m)$: TRUE.

(f) $\forall m \exists x \left(\frac{x}{|x|+1} \geq m \right)$: FALSE.

(g) $\exists x \exists y (x^2 + y^2 < xy)$: FALSE.

Sets and Numbers

16. $FTF \quad FTF \quad TFT$.

17. $LHS - RHS = \frac{1}{2} [(a-b)^2 + (b-c)^2 + (c-a)^2]$.

18. (a) See EXAMPLE 2.1.5.

(b) $RHS - LHS = (a_1 b_2 - a_2 b_1)^2$.

(c) $RHS - LHS \geq 0$.

19. (a) $B \subseteq A$.

(b) $B \subseteq A$.

(c) $A = B$.

(d) $A \subseteq B$.

(e) $A \cap B = \emptyset$.

(f) $A = B = \emptyset$.

(g) $A = B$.

20. (a) No.

(b) No.

(c) Yes.

21. (a) $\{1, 4\}$.

(b) Use set identities.

(c) $B = \emptyset$.

22. (a) TRUE.

(b) FALSE. For example, if $A = \emptyset$ and $B = \{1\}$, then $|A \setminus B| = 0 \neq 0 - 1 = |A| - |B|$.

(c) FALSE. For example, if $A = \{1, 2\}$ and $B = \{1, 3\}$, then $|A \cup B| = 3 \neq 2 + 2 = |A| + |B|$.

(d) TRUE.

(e) FALSE. Observe that $(5, 6) \notin A \times B \iff 5 \notin A$ or $6 \notin B$ and use the fact that $p \vee q$ does *not* imply $p \wedge q$. Counterexample : $A = B = \{5\}$.

(f) TRUE (p does logically imply $p \vee q$).

(g) TRUE.

(h) TRUE.

23. Use set identities.

24. Use the definition of *divisibility*. (For instance, $a | b \iff b = ak \Rightarrow bc = a(ck) \iff a | bc$.)

25. (a) No.

(b) Yes.

(c) No.

(d) No.

26. (a) 2 and 5.

(b) -10 and 9.

(c) 77 and 0.

(d) 0 and 0.

(e) -1 and 4.

27. (a) $39 = 3 \cdot 13$.

(b) $81 = 3^4$.

(c) $101 = 101$ (prime number).

(d) $289 = 17^2$.

(e) $899 = 29 \cdot 31$.

28. (a) 6.

(b) 3.

(c) 11.

(d) 1.

29. (a) $1 = (-1) \cdot 10 + 1 \cdot 11$.

(b) $1 = 21 \cdot 21 + (-10) \cdot 44$.

(c) $12 = (-1) \cdot 36 + 1 \cdot 48$

(d) $1 = 13 \cdot 55 + (-21) \cdot 34$

(e) $3 = 11 \cdot 213 + (-20) \cdot 117$.

(f) $223 = 1 \cdot 0 + 1 \cdot 223$.

30. (a) FALSE. (The integers are *not* necessarily positive.)

(b) FALSE.

(c) TRUE. (One implication is immediate.)

Functions

32. (a) Injective but not surjective.

(b) Injective but not surjective.

- (c) Surjective but not injective.
 (d) Surjective but not injective.
 (e) Neither injective nor surjective.
 (f) Bijective : $k^{-1} = k$.
 (g) Bijective.
 (h) Neither injective nor surjective.
 (i) Bijective : $n^{-1}(x) = \sqrt[4]{x}$.
 (j) Bijective.
 (k) Injective but not surjective.
 (l) Bijective : $w^{-1} = w$.
- 33.** (a) *Eight* functions : $f_1 = \{(1, 4), (2, 4), (3, 4)\}$, $f_2 = \{(1, 4), (2, 4), (3, 5)\}$, etc. None is one-to-one but *six* are onto.
 (b) *Nine* functions. *Six* are one-to-one but none is onto.
 (c) *Four* functions. *Two* are one-to-one and onto.
- 34.** (a) $n \mapsto n + 1$.
 (b) $0 \mapsto 0$, $n \mapsto n - 1$.
 (c) $2k \mapsto 2k + 1$, $2k + 1 \mapsto 2k$.
 (d) $2k \mapsto 2k + 1$, $2k + 1 \mapsto 2k + 1$.
- 35.** (a) No. Counterexample : $f(x) = x$ and $g(x) = -x$.
 (b) No. Counterexample : $f(x) = g(x) = x$.
 (c) Yes. Prove that $(f \circ g)(x_1) = (f \circ g)(x_2) \Rightarrow x_1 = x_2$.
 (d) No. Give a counterexample.
 (e) No. Give a counterexample.
 (f) Yes. Prove that $\forall z \in \mathbb{R}, \exists x \in \mathbb{R}$ such that $(f \circ g)(x) = z$.
- 36.** $ad + b = bc + d$.
- 37.** (a) $(1, 3, 5, 2, 4)$.
 (b) $(1, 2, 4)(3, 6, 5)$.
 (c) $(1, 2, 3, 4, 5, 6)$.

(d) $(1, 3, 5)(2, 4, 6)$.(e) $(1, 6, 5, 4, 3, 2)$.(f) $(1)(2)(3)(4)(5)$.(g) $(1)(2, 3, 4, 5)$.**38.** (a) $(1, 4, 7, 6, 9, 3, 2, 5, 8)$.(b) $(1, 3, 6, 9, 8, 2, 5, 4, 7)$.(c) $(1)(2, 4)(3, 5)(6, 8)(7, 9)$.(d) $(1)(2, 5, 4, 3)(6, 9, 8, 7)$.(e) $(1, 8, 6, 4, 2, 9, 7, 5, 3)$.(f) $(1)(2)(3)(4)(5)(6)(7)(8)(9) = \iota$.(g) $(1, 9)(2, 8)(3, 5)(4, 6)(7)$.**39.** (a) $a = 3, b = 5, c = 4, d = 1, e = 2$.(b) $(1, 5, 2)(3, 4) = (1, a, b)(3, c) \Rightarrow a = 5, b = 2, c = 4$.**40.** (a) $(1)(2)(3) = \iota, (1, 2) = (1, 2)(3), (1, 3) = (1, 3)(2), (2, 3) = (1)(2, 3), (1, 2, 3), (1, 3, 2)$.(b) $(1)(2)(3)(4) = \iota, (1, 2), (1, 3), (1, 4), (2, 3), (2, 4), (3, 4), (1, 2, 3), (1, 3, 2), (1, 2, 4), (1, 4, 2), (1, 3, 4), (1, 4, 3), (2, 3, 4), (2, 4, 3), (1, 2, 3, 4), (1, 4, 3, 2), (1, 3, 2, 4), (1, 4, 2, 3), (1, 2, 4, 3), (1, 3, 4, 2), (1, 2)(3, 4), (1, 3)(2, 4), (1, 4)(2, 3)$.**41.** (a) FALSE. Give a counterexample.(b) FALSE : $(i, j)(j, k) = (i, j, k) \neq (i, k, j) = (j, k)(i, j)$.

(c) FALSE. Give a counterexample.

(d) TRUE.

42. (a) Multiply both sides (from the right) by β^{-1} .(b) $\alpha\beta = \alpha\gamma \Rightarrow (\alpha^{-1}\alpha)\beta = (\alpha^{-1}\alpha)\gamma \Rightarrow \beta = \gamma$.**43.** • $(1, 3, 2)$.• $(1, 4, 3, 2)$.• $(1, 5, 4, 3, 2)$.

- $(1, n, n - 1, \dots, 3, 2)$.

- 44.**
- $(2, 4)(2, 3)$.
 - $(2, 8)(2, 6)(2, 4)$.
 - $(1, 2)(3, 5)(3, 4)(6, 9)(6, 8)(6, 7)$.
 - $(1, 5)(1, 6)(2, 9)(3, 4)(3, 8)$.

- 45.**
- (a) Even.
 - (b) Odd.
 - (c) Odd.
 - (d) Odd.

Mathematical Induction

46. (a) $3(2^{11} - 1)$.

(b) $\frac{1}{2}(3^{11} + 1)$.

(c) $\frac{1}{2}(3^{11} - 2^{12} + 1)$.

(d) $3^{11} + 3 \cdot 2^{11} - 4$.

(e) 138.

47. (a) 500 497.

(b) 9 150.

(c) $\frac{k^2-k-2}{2}$.

(d) $2^{26} - 1$.

(e) $\frac{3}{2}(3^n - 1)$.

(f) $2\left(1 - \frac{1}{2^{n+1}}\right)$.

(g) $\frac{1}{3}(1 + (-1)^n \cdot 2^{n+1})$.

48. (b) $\frac{n}{n+1}$.

(c) $\frac{1}{4} - \frac{1}{2(n+1)(n+2)}$.

(d) $1 - \frac{1}{(n+1)!}$.

56. (a) i. $\sum_{k=1}^n (3k) = \frac{3n(n+1)}{2}$.

ii. $\sum_{k=1}^n (2k - 1) = n^2$.

- iii. $\sum_{k=1}^n \frac{1}{k(k+1)} = \frac{n}{n+1}$.
- iv. $\sum_{k=1}^n (4k - 3) = n(2n - 1)$.
- v. $\sum_{k=1}^n (3k + 1)(3k + 4) = 3n(n + 1)(n + 3) + 4n$.

Counting

- 61.** (a) $9 + 9 \cdot 9 + 9 \cdot 9 \cdot 8 = 738$.
 (b) $1999 - 199 = 1800$.
 (c) $5 \cdot 9 \cdot 9 \cdot 9 \cdot 9 = 32\,805$.
- 62.** (a) $36^4 = 1\,679\,616$.
 (b) $P(36, 4) = 36 \cdot 35 \cdot 34 \cdot 33 = 1\,413\,720$.
- 63.** (a) $7! = 5\,040$.
 (b) $6! = 720$.
 (c) $2 \cdot 6! = 1\,440$.
 (d) $7! - 2 \cdot 5! = 4\,800$.
 (e) $2 \cdot 5! = 240$.
 (f) $6! = 720$.
 (g) $5 \cdot 6! = 3\,600$.
 (h) $7! - 2 \cdot 6! = 3\,600$.
- 64.** (a) 60.
 (b) 40.
 (c) 14.
- 65.** (a) $30!$.
 (b) $\binom{30}{4}$.
 (c) $\binom{10}{3} \binom{15}{7}$.
 (d) $15! \cdot 5! \cdot 10!$.
- 66.** (a) $8^5 = 32\,768$.
 (b) $8^5 - (8)_5 = 26\,048$.
 (c) $3^5 - \binom{3}{1}2^5 + \binom{3}{2}1^5 = 150$.

(d) $2^{10} = 1\,024$.

67. (a) $7^5 = 16\,807$.

(b) $9\,031$.

(c) $4\,380$.

(d) $P(7, 5) = 2\,520$.

(e) $1\,800$.

(f) $2\,401$.

68. (a) $P(9, 3) = 504$.

(b) $P(8, 4) = 1\,680$.

69. (a) $\binom{12}{3} + \binom{12}{5} = 748$.

(b) $\binom{14}{5} - \binom{12}{3} = 1\,782$.

70. (a) $\binom{5}{3}\binom{7}{2} = 210$.

(b) $\binom{12}{5} - \binom{7}{5} = 771$.

(c) $\binom{5}{0}\binom{7}{5} + \binom{5}{1}\binom{7}{4} = 196$.

71. (a) $\binom{16}{9} = 11\,440$.

(b) $2^{16} - 1 = 65\,535$.

72. (a) $\binom{9}{2} = 36$.

(b) $36 - 8 = 28$.

(c) $\binom{9}{3} = 84$.

(d) $\binom{9}{3} - \binom{8}{2} = 56$.

73. (a) $32 - 80x + 80x^2 - 80x^3 + 10x^4 - x^5$.

(b) $64a^6 - 576a^5b + 2160a^4b^2 - 4320a^3b^3 + 4860a^2b^4 - 2916ab^5 + 729b^6$.

(c) $a^6 + 12a^5 + 60a^4 + 160a^3 + 240a^2 + 192a + 64$.

(d) $x^5 - 15x^3 + 90x - \frac{270}{x} + \frac{405}{x^3} - \frac{243}{x^5}$.

(e) $x^{14} + 7x^{11} + 21x^8 + 35x^5 + 35x^2 + \frac{21}{x} + \frac{7}{x^4} + \frac{1}{x^7}$.

74. (a) $\binom{12}{5} = 792$.

(b) $\binom{11}{5}2^5(-1)^6 = 14\,784$.

- (c) 0.
- (d) $-252a^5b^5$.
- (e) 16.
- (f) 405.
- (g) $64y^6$.

Recursion

- 77.** (a) $a_n = 5a_{n-1}$, $a_0 = 1$ (or $a_1 = 5$).
 (b) $b_n = b_{n-1} + n$, $b_1 = 1$.
 (c) $c_n = -c_{n-1}$, $c_0 = 1$ (or $c_1 = -1$).
 (d) $\delta_n = \delta_{n-1}$, $\delta_0 = \sqrt{2}$ ($\delta_1 = \sqrt{2}$).
 (e) $e_n = e_{n-1} + (-1)^{n+1}$, $e_0 = 0$.
 (f) $f_n = \frac{f_{n-1}}{1+f_{n-1}}$, $f_1 = \frac{1}{2}$.
 (g) $g_n = g_{n-1} + 10^{-n}$, $g_1 = 0.1 (= 10^{-1})$.
- 78.** (a) $a_n = \frac{1}{n+1}$.
 (b) $b_n = 5 \cdot 2^{n-1} - 3$.
 (c) $c_n = n(n+2)$.
 (d) $d_n = \frac{1}{3}(2^{n+1} + (-1)^n)$.
 (e) $e_n = \frac{1}{4}(3^{n+1} - 2n - 3)$.
 (f) $f_n = 1^2 + 2^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$.
 (g) $g_n = 5n!$.
- 79.** (a) $s_n = \lfloor \frac{n+1}{2} \rfloor = \begin{cases} k & \text{if } n = 2k \\ k+1 & \text{if } n = 2k+1. \end{cases}$
 (b) $t_n = 2^{F_n}$, where F_n is the n^{th} term of the Fibonacci sequence.
- 80.** We are letting P_n be the population (in billions) n years after 1995.
 (a) $P_n = 1.03 \cdot P_{n-1}$, $P_0 = 7$.
 (b) $P_n = 7(1.03)^n$.
 (c) $P_{15} = 7(1.03)^{15} \approx 10.9$ billions.

- 81.** (a) $s_n = s_{n-1} + s_{n-2} + s_{n-3} + 2^{n-3}$.
 (b) $s_0 = s_1 = s_2 = 0$.
 (c) $s_7 = 47$.
- 82.** (a) $a_n = a_{n-1} + a_{n-2}$.
 (b) $a_0 = 1, a_1 = 1$.
 (c) $a_{10} = 89$.
- 83.** (a) $a_n = \frac{1}{1.08}a_{n-1}$.
 (b) $a_{20} = \left(\frac{1}{1.08}\right)^{20} \approx 0.215$.
 (c) $a_{80} = \left(\frac{1}{1.08}\right)^{80} \approx 0.002$.
 (d) $\left(\frac{1}{1.1}\right)^{20} \approx 0.148$.
 (e) $\left(\frac{1}{1.1}\right)^{80} \approx 0.0004$.
- 84.** (a) $a_n = -9 \cdot 2^{n-1} + 4 \cdot 3^{n-1} + n + 4$.
 (b) $b_n = -7 \cdot 2^{n+2} + 7n \cdot 2^{n+1} + n^2 + 8n + 20$.
 (c) $c_n = \frac{1+(-1)^n}{2^{n+1}}$.
 (d) $d_n = 1 + (-1)^n + \frac{100}{99} \cdot 10^n$.
 (e) $e_n = 1 + \frac{n(n+1)}{2}$.
 (f) $L_n = \left(\frac{1+\sqrt{5}}{2}\right)^n + \left(\frac{1-\sqrt{5}}{2}\right)^n$.
 (g) $f_n = (n^2 - n + 2) \cdot 2^{n-1}$.
 (h) $a_n = C \cdot (-3)^n + n^2 + n + \frac{2^{n+1}}{5}$.
- 85.** (a) $y(n) = y(0)(1+r)^n$.
 (b) $y(n) = y(0) \left(1 + \frac{i}{100}\right)^n$.
 (c) $y(50) = 117\,391$.
 (d) $y(50) = 184\,565$.
- 86.** (a) $y(n) = \left(y(0) + \frac{d}{r}\right)(1+r)^n - \frac{d}{r}$.
 (b) $y(50) = 1\,281\,299$.
 (c) $y(50) = 1\,853\,336$.
- 87.** (a) The recurrence relation $y(n+1) = (1+r)y(n) + d(1+i)^n$ yields
 $y(n) = y(0)(1+r)^n + d \frac{(1+r)^n - (1+i)^n}{r-i}$ for $i \neq r$, and
 $y(n) = y(0)(1+r)^n + nd(1+r)^{n-1}$ for $i = r$.

(b) $y(50) = 19\,442\,723.$

88. (a) $y(n) = y(0)(1+r)^n + \frac{d}{r}[1 - (1+r)^n].$

(b) $d = 1\,028.61.$

89. (b) i. $d > p_0 r.$

ii. $d = p_0 r.$

iii. $d < p_0 r.$

90. (a) $s_n = \frac{d}{r}[1 - (1-r)^n].$

(c) $s_n \rightarrow \frac{d}{r} = 400.$

Linear Equations and Matrices

91. (a) $k = 10.$

(b) $k \neq 10.$

(c) Infinitely many values.

92. (a) $x_1 = 3, x_2 = -2, x_3 = 4.$

(b) $x_1 = 2\alpha - 3\beta + 4, x_2 = \alpha, x_3 = 3 - 4\beta, x_4 = \beta$

(c) $(3, -2, 4, -1).$

93. (a) (i) none ; (ii) $k \neq 2$; (iii) $k = 2.$

(b) (i) $k \neq 4$; (ii) none ; (iii) $k = 4.$

(c) (i) every value : $k \in \mathbb{R}$; (ii) none ; (iii) none.

(d) (i) none ; (ii) $k \neq 11$; (iii) $k = 11.$

94. The system has *either* no solution (when $2a + 3b - c \neq 0$) *or* infinitely many solutions (when $2a + 3b - c = 0$).

95. $3a + b = c.$

97. (a) There are infinitely many choices (for example, $r = 1, s = 0$).

(b) $a = 3, b = 1, c = 8, d = -2.$

98. (a) TRUE.

(b) FALSE. (This *equality* is an *identity* if and only if $AB = BA.$)

(c) TRUE.

- (d) FALSE.
 (e) FALSE. (This *equality* is an *identity* if and only if $AB = BA$.)
 (f) TRUE.
 (g) FALSE. (This condition is equivalent to $AB = BA$.)
 (h) TRUE.
 (i) TRUE.
 (j) TRUE.

99. (b) $\text{tr}(AB - BA) = 0 \neq 2 = \text{tr}(I_2)$.

100. (a) TRUE.

- (b) FALSE. Give a counterexample.

102. $A^2 = \begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix}$, $A^3 = \begin{bmatrix} 14 & 13 \\ 13 & 14 \end{bmatrix}$, $A^4 = \begin{bmatrix} 41 & 40 \\ 40 & 41 \end{bmatrix}$, $A^5 = \begin{bmatrix} 122 & 121 \\ 121 & 122 \end{bmatrix}$.

103. (a) There is no such matrix.

- (b) $\begin{bmatrix} 0 & b \\ 1/b & 0 \end{bmatrix}$.
 (c) $\begin{bmatrix} 0 & b \\ -1/b & 0 \end{bmatrix}$.
 (d) $\begin{bmatrix} -2\alpha & -2\beta \\ \alpha & \beta \end{bmatrix}$.
 (e) $\begin{bmatrix} \alpha & 0 \\ 0 & \beta \end{bmatrix}$.
 (f) $\begin{bmatrix} \alpha & \beta \\ 0 & \alpha + \beta \end{bmatrix}$.
 (g) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$.

104. (a) $A^{-1} = \begin{bmatrix} 3 & -2 \\ -4 & 3 \end{bmatrix}$, $x = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$.

(b) $A^{-1} = \frac{1}{4} \begin{bmatrix} 7 & -9 \\ -5 & 7 \end{bmatrix}$, $x = \begin{bmatrix} 3/4 \\ 1/4 \end{bmatrix}$.

105. (a) $\begin{bmatrix} -5 & -2 & 5 \\ 2 & 1 & -2 \\ -4 & -3 & 5 \end{bmatrix}$.

(b) $\frac{1}{6} \begin{bmatrix} -21 & 11 & 8 \\ 9 & -5 & -2 \\ -3 & 3 & 0 \end{bmatrix}$.

(c) $\begin{bmatrix} 0 & 1 & 0 & 0 \\ -2 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & -3 & -2 & 1 \end{bmatrix}$.

Determinants

106. (a) Invertible.

(b) Not invertible.

(c) Invertible if and only if $abc \neq 0$.

(d) Not invertible.

(e) Invertible.

107. (a) $\lambda_1 = 1, \lambda_2 = 3$.

(b) $\lambda_1 = 3, \lambda_2 = 8$.

(c) $\lambda_1 = 1, \lambda_2 = 2, \lambda_3 = 3$.

(d) $\lambda_1 = -1, \lambda_2 = 0, \lambda_3 = 2$.

(e) $\lambda_1 = -4, \lambda_2 = 0, \lambda_3 = 2$.

108. (a) $\alpha \in \mathbb{R}$.

(b) $\alpha \in \mathbb{R} \setminus \{0, 1\}$.

109. (a) 8.

(b) 60.

(c) 4.

(d) -210.

(e) 120.

(f) 60.

110. (a) 0.

(b) 25.

(c) 30.

(d) 40.

(e) 10.

111. Use properties of the determinants.

112. (a) -6.

(b) -37.

(c) -74.

(d) 39.

(e) 18.

(f) 98.

113. (a) $\det(A^T A) = 9$.

(b) The number $\det(A^T A)$ is positive.

115. All statements are TRUE.

116. (a) $\left(\frac{a}{a^2+b^2}, -\frac{b}{a^2+b^2} \right)$.

(b) $(-\frac{3}{5}, -1, \frac{4}{5})$.

(c) $(-\frac{1}{7}, \frac{9}{14}, \frac{2}{7})$.

(d) $(\frac{9}{37}, -\frac{7}{37}, -\frac{8}{37})$.

117. (a) $x = 5u - 8v$ and $y = 5v - 3u$.

(b) $x = u \cos \theta + v \sin \theta$ and $y = v \cos \theta - u \sin \theta$.

119. (a) $\frac{1}{10} \begin{bmatrix} 2 & 4 & 2 \\ -5 & 0 & -10 \\ -6 & -2 & -6 \end{bmatrix}$.

(b) $\frac{1}{35} \begin{bmatrix} -15 & 25 & -26 \\ 10 & -5 & 8 \\ 15 & -25 & 19 \end{bmatrix}$.

$$(c) \frac{1}{37} \begin{bmatrix} -21 & -1 & -13 \\ 4 & 9 & 6 \\ -6 & 5 & -9 \end{bmatrix}.$$

120. *FTT TFT.*

Vectors, Lines, and Planes

121. (b) $(-5, 7)$.

(c) $(-1, -4)$.

122. $a = 3, b = -1$.

$$\mathbf{123.} \quad \vec{u} + \vec{v} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad \vec{u} - \vec{v} = \begin{bmatrix} 1 \\ -3 \end{bmatrix}, \quad 3\vec{u} - 2\vec{v} = \begin{bmatrix} 3 \\ -7 \end{bmatrix}, \quad \| -2\vec{u} \| = 2\sqrt{2}, \quad \| \vec{u} + \vec{v} \| = \sqrt{2}.$$

125. (a) $\theta = 135^\circ$.

(b) $\theta = 180^\circ$.

126. (a) $r = 2$.

(b) $k = \frac{8}{5}$.

128. (a) $\cos \theta = -\frac{13\sqrt{10}}{50} \approx -0.82219 \Rightarrow \theta \approx 145.3^\circ$.

(b) $\cos \theta = -\frac{\sqrt{19}}{57} \approx -0.07647 \Rightarrow \theta \approx 94.4^\circ$.

129. $(w_2 + w_3)\vec{i} - w_1\vec{j} - w_1\vec{k}; \quad (w_1 + w_2 + w_3)(\vec{j} - \vec{k}); \quad (w_1 + w_2 + w_3)(\vec{j} - \vec{k}); \quad -2w_1 + w_2 + w_3; \quad -2w_1 + w_2 + w_3$.

$$\mathbf{130.} \quad (a) \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}.$$

$$(b) \begin{bmatrix} -2 \\ 17 \\ 7 \end{bmatrix}.$$

131. (a) $\angle A \approx 78.65^\circ, \angle B \approx 64.29^\circ, \angle C \approx 37.06^\circ$.

(b) $\frac{\sqrt{478}}{2}$.

(c) $\frac{3\sqrt{10}}{2}$.

(d) $5\sqrt{6}$.

(e) 1.

133. $\vec{r} = \begin{bmatrix} 2 \\ 1 \\ 8 \end{bmatrix} + t \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}; \quad x = 2 + 2t, y = 1 + 3t, z = 8 + 4t.$

135. $x = 2t, y = 2 + 3t, z = -1 - 7t.$

136. $17x - 6y - 5z = 32.$

137. $x + y - z = 13.$

138. $6x + y + z = 23.$

139. $(-1, 2, -3).$

140. $49x - 7y - 25z + 61 = 0.$

141. 3.

142. $(1, 1, 1).$

143. $\vec{r} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}.$

144. $\vec{r} = t \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}.$

145. (a) 3.

Complex Numbers

146. $i, -i, -i.$

147. (a) $4 + 5i.$

(b) $-7 + 24i.$

(c) $-1 + i.$

(d) 1.

(e) 3.

(f) 3.

148. (a) 7, 7, 0, 7.

(b) $2i, 0, -2, 2$.

(c) $16 - 30i, -16, 30, 34$.

(d) $-\frac{7}{41} - \frac{22}{41}i, -\frac{7}{41}, \frac{22}{41}, \frac{1}{41}\sqrt{533}$.

149. HINT : If $z = x+iy$, $z = u+iv \in \mathbb{C}$ then $\frac{z}{w} \in \mathbb{R} \iff z\bar{w} \in \mathbb{R} \iff xv = yu$.

152. (a) $2(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3}) = 2e^{i\frac{\pi}{3}}$.

(b) $\sqrt{2}(\cos \frac{5\pi}{4} + i \sin \frac{5\pi}{4}) = \sqrt{2}e^{i\frac{5\pi}{4}}$.

(c) $5\sqrt{2}(\cos \frac{4\pi}{4} + i \sin \frac{3\pi}{4}) = 5\sqrt{2}e^{i\frac{3\pi}{4}}$.

(d) $\cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3} = e^{i\frac{2\pi}{3}}$.

153. $z = \ln \sqrt{2} + i(\frac{\pi}{4} + 2k\pi)$, $k \in \mathbb{Z}$.

154. $\cos 1 - i \sin 1$, $\cos(\ln 2) + i \sin(\ln 2)$, $\pm \frac{1}{\sqrt{2}}(1+i)$.

158. HINT : Evaluate $C + iS$. Use *de Morgan's formula* and **Exercise 191**.

159. (a) $-\frac{1}{5} \pm i\frac{7}{5}$.

(b) $2 - 3i, 1 + i$.

(c) 1, ω , ω^2 , ω^3 and ω^4 , where $\omega = e^{i\frac{3\pi}{5}}$.

(d) $\frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}} - i\frac{1}{\sqrt{2}}$.

(e) $2^{\frac{1}{6}} e^{i\frac{\pi}{4}}, 2^{\frac{1}{6}} e^{i\frac{11\pi}{12}}, 2^{\frac{1}{6}} e^{i\frac{19\pi}{12}}$.

(f) $e^{i\frac{(8k\pm 3)\pi}{12}}$, $k = 0, 1, 2$.

(g) $12^{\frac{1}{10}} e^{i\frac{(6k-1)\pi}{15}}$ and $12^{\frac{1}{10}} e^{i\frac{(6k-2)\pi}{15}}$, $k = 0, 1, 2, 3$.

(h) 1, 1, 2, $-1 \pm i$.