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Lecture Notes CCR

MAT 102 - Discrete Mathematics

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Mathematics is not about calculations but about ideas. [...] Calculations are merely a means to an end. [...] Not all ideas are mathematics; but all good mathematics must contain an idea. [...] There are [...] at least five distinct sources of mathematical ideas. They are ***number****,* ***shape****,* ***arrangement****,* ***movement****, and* ***chance****. [...] The driving force of mathematics is* ***problems****. [...] Another important source of mathematical inspiration is* ***examples****.*

IAN STEWART

It is not easy to say what mathematics is, but "I know it when I see it" is the most likely response of anyone to whom this question is put. The most striking thing about mathematics is that it is very different to science, and this compounds the problem of why it should be found so useful in describing and predicting how the Universe works. Whereas science is like a long text that is constantly being redrafted, updated, and edited, mathematics is entirely cumulative. Contemporary science is going to be proven wrong, but mathematics is not. The scientists of the past were well justified in holding naïve and erroneous views about physical phenomena in the context of the civilizations in which they lived, but there can never be any justification for establishing erroneous mathematical results. The mechanics of ARISTOTEL *is wrong, but the geometry of* EUCLID *is, was, and always will be correct. Right and wrong mean different things in science and mathematics. In the former, "right" means correspondence with reality; in mathematics it means logical consistency.*

JOHN D. BARROW

Mathematics is a way of representing and explaining the Universe in a symbolic way.

John D. Barrow

Mathematics may be defined as the subject in which we never know what are we talking about, nor whether what we are saying is true.

Bertrand Russell

What is mathematics, anyway ?

In a broad sense, mathematics include all the related areas which touch on *quantitative, geometric*, and *logical themes.* This includes Statistics, Computer Science, Logic, Applied Mathematics, and other fields which are frequently considered distinct from mathematics, as well as fields which study the study of mathematics (!) – History of Mathematics, Mathematics Education, and so on. We draw the line only at experimental sciences, philosophy, and computer applications. Personal perspectives vary widely, of course. Probably the only absolute definition of mathematics is : *that which mathematicians do.*

Contrary to common perception, mathematics does not consists of "crunching numbers" or "solving equations". There are branches of mathematics concerned with *setting up* equations, or *analyze* their solutions, and there are

parts of mathematics devoted to *creating methods* for doing computations. But there are also parts of mathematics which have nothing at all to do with numbers and equations.

The current mathematics literature can be divided, roughly, into two parts : *"pure" mathematics* (i.e. mathematics for mathematics) and *"applied" mathematics* (i.e. mathematics for something else).
The first group divides roughly into just a few broad overlapping areas :

- **Foundations** : considers questions in logic or set theory – the very language of mathematics.

- **Algebra** : is principally concerned with symmetry, patterns, discrete sets, and the rules for manipulating arithmetic operations (one might think of this as the outgrowth of arithmetic and algebra classes in primary and secondary school).

- **Geometry** : is concerned with shapes and sets, and the properties of them which are preserved under various kinds of transformations (this is related to elementary geometry and analytic geometry).

- **Analysis** : studies functions, the real number line, and the ideas of continuity and limit (this is the natural succesor to courses in graphing, trigonometry, and calculus).

The second broad part of mathematics literature includes those areas which could be considered either independent disciplines or central parts of mathematics, as well as areas which clearly use mathematics but are interested in non-mathematical ideas too :

- **Probability and Statistics** : has a dual nature – mathematical and experimental.

- **Computer Sciences** : consider algorithms and information handling.

- **Application to Sciences** : significant mathematics must be developped to formulate ideas in the **physical sciences** (e.g. mechanics, optics, electromagnetism, relativity, astronomy, etc.), **engineering** (e.g. control, robotics, etc.), and other branches of **science** (e.g. biology, economics, social sciences).

The division between mathematics and its applications is of course vague.

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